Reserved

217 McHlaine hell

69%

September 21, 2000   MATHEMATICS 110 (31)   Total Marks - 18
                                   Quiz #1   Time: 45 minutes

Last Name: ____ ___ ___ ___ _____   Student Number: ___ _ __ _

[5] 1. **SHORT ANSWER SECTION** (Answers will be marked either RIGHT or
        WRONG.)
        Evaluate the following EXACTLY:

        a) $\cos(\pi/3) =$ $\frac{\pi}{3} = \frac{180°}{3} = 60°$ $\cos 60° = \cos\frac{1}{2}$ $\boxed{=\frac{-1}{2}}$

        b) $\tan(11\pi/4) =$ $\frac{11(180°)}{4} = 4\sqrt{198°}$ $\frac{495°}{360°}$ $\frac{495}{135}$ $180-135=45°$ $\tan 45° = \frac{-1}{1}$ $\boxed{(-1)}$

        c) $\sin(5\pi/12) =$ $75°$ $\sin 45+30 = \frac{\sin 45 \cos 30 + \cos 45 \sin x}{\frac{1}{\sqrt{2}} \frac{\sqrt{3}}{2} \quad \frac{1}{\sqrt{3}} \frac{1}{2}}$ $\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \boxed{\frac{1+\sqrt{3}}{2\sqrt{2}}}$

        State whether the following statements are TRUE or FALSE:

     (T) d) The relation given by $x + 2y - 6 = 0$ defines a function f with $y = f(x)$
            and domain $(-\infty, \infty)$. $\frac{2y}{2} = \frac{6-x}{2}$ $y = \frac{-x}{2} + 3$

        e) The relation given by $x + 2y^2 - 6 = 0$ defines a function f with $y = f(x)$
     (T)   and domain $(-\infty, 6)$. $\frac{2y^2}{2} = \frac{6-x}{2}$ $y = \sqrt{\frac{-x}{2} + 3}$

        f) The relation given by $x + 2\sqrt{y} - 6 = 0$ defines a function f with $y = f(x)$
     (T)   and domain $(-\infty, 6)$. $2\sqrt{y} = 6-x$ $y = (12 - 2x)^2$ $\sqrt{y} = 12 - 2x$

        Find the following:
        g) The slope of a line through the point $(0,1)$ and parallel to the line $y =$
           $-3x + 2$. $m = \frac{y_1 - y_2}{x_1 - x_1} = \frac{-1 - -4}{1 - 2} = \frac{3}{-1}$ $\boxed{-3}$

        h) The slope of a line through the points $(0,1)$ and $(-1,1)$.
           $m = \frac{y_1 - y_2}{x_1 - x_2}$ $m = \frac{1-1}{0+1}$ $m = \frac{0}{1}$ (This is a horizontal line and the slope is 0)

        i) The domain of the function $f(x) = x^3 - 3x^5 + 7 - x$.

        j) The domain of the function $f(x) = \sin(\sqrt{x})$.

[5]

2. Solve the following inequalities and express your answer using interval notation. Show all your work.

a) $-3 \leq 5 - 2x \leq 2$

$$-3 \leq 5 - 2x$$
$$0 \leq 8 - 2x$$
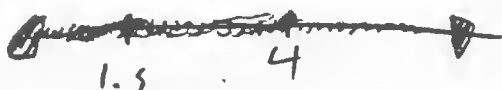$$4 \leq x$$

$-3 \leq 5 - 2x \qquad 5 - 2x \leq 2$

$\cdot\ 5 - 2x \geq 2 \quad \dfrac{-8 \leq -2x}{-2} \dfrac{}{-2}$

$3 - 2x \geq 0 \quad 4 \geq x$
$x \geq 1.5$

$\dfrac{-2x}{-2} \leq \dfrac{-}{} $

$x \geq 1.$

$[1.5, 4]$

b) $\dfrac{x^2+1}{3-2x} < 2$ $\qquad \dfrac{18}{-5} \quad \dfrac{5}{-5}$

$\dfrac{x^2+1}{3-2x} - \dfrac{2}{1} < 0$

$\dfrac{x^2+1-6+4x}{3-2x} < 2$

$\dfrac{x^2+4x-5}{3-2x} < 0$

$\dfrac{(x-5)(x+1)}{3-2x} < 0$
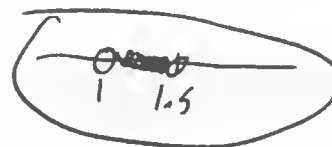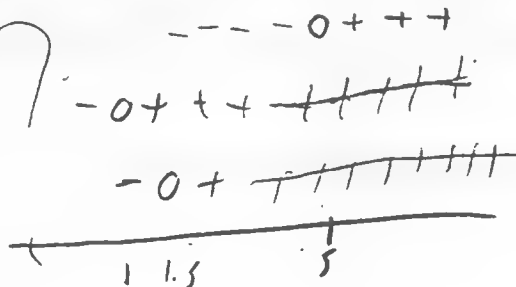
$(-\infty, 1)$

$x - 5 < 0$
$x + 1 < 0$

$3 - 2x > 0$
$\dfrac{x^2+4x-5}{3-2x} < 0$

[4]   3. Consider the following function:

$$f(x) = 1 - |2 - x|$$

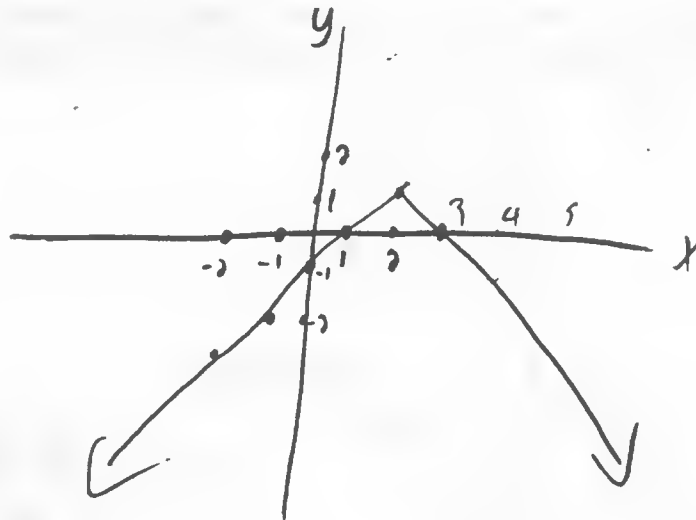a) Give a piecewise defined formula for $f(x)$ which does not involve absolute values.

$$1 = |2-x| \begin{cases} 2-x & \text{if } 2-x \geq 0 \\ -2+x & \text{if } 2-x < 0 \end{cases}$$

$$2-x \quad \text{if } x \geq 2$$

$$-(2+x) \quad \text{if } x < 2$$

b) Sketch the graph of $y = f(x)$.

x	y
-1	-2
0	-1
1	0
2	1
-2	-3
3	0
4	-1

c) Find the domain and range of $f(x)$.   looked at graph

$$D\{x \in \mathbb{R}\}$$
$$R\{y \in \mathbb{R} \mid y \leq 1\}$$

[2]   4. Prove or disprove that the function $f(x) = -3(x-2)^2 + 6$ is an even function.

$$f(x) \neq f(-x)$$

$3(x-2)^2 + 6$	$-3(-x-2)^2 + 6$
$(x^2 - 4x + 4) + 6$	$-3(x^2 + 4x + 4) + 6$
$3x^2 + 12x - 12 + 6$	$-3x^2 - 12x - 12 + 6$
$3x^2 + 12x - 6$	$= -3x^2 - 12x - 6$

$$LS \neq RS$$

$\therefore$ this is not an even function

[2]   5. Find an equation for the line which passes through the point $(0, 5/6)$ and is
perpendicular to the line $x + 2y = 6$ and then sketch its graph.

find 2 points from $x + 2y = 6$

$$2y = \frac{6-x}{2}$$

$$y = 3 - \frac{x}{2}$$

x	y
4	1
2	2

$m = \frac{y_1 - y_2}{x_1 - x_2}$

$= \frac{1-2}{4-2}$

$m = \frac{-1}{2}$

Find an equation
$P_1 = (0, \frac{5}{6})$
$GP = (x, y)$
$m = \frac{-1}{2}$

$m = \frac{y_1 - y_2}{x_1 - x_2}$

$\frac{-1}{2} = \frac{y - \frac{5}{6}}{x - 0}$

$-x = 2y - \frac{10}{6}$

$\frac{\frac{10}{6} - x}{2} = \frac{2y}{2}$
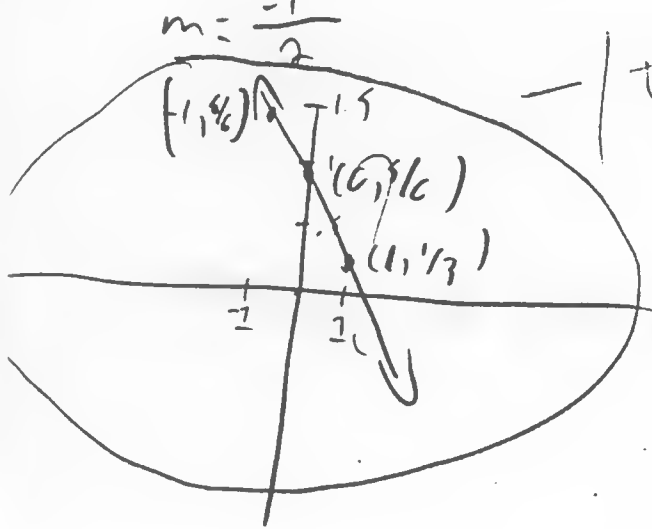
$\boxed{\frac{5}{6} - \frac{x}{2} = y}$

x	y
1	1/3
0	5/6
-1	4/6

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Last Name: _ _ _____ _____   Student Number: _ ____ ____

8 [10]   1. Consider the following graph of the function f on the interval $[-2,12]$:
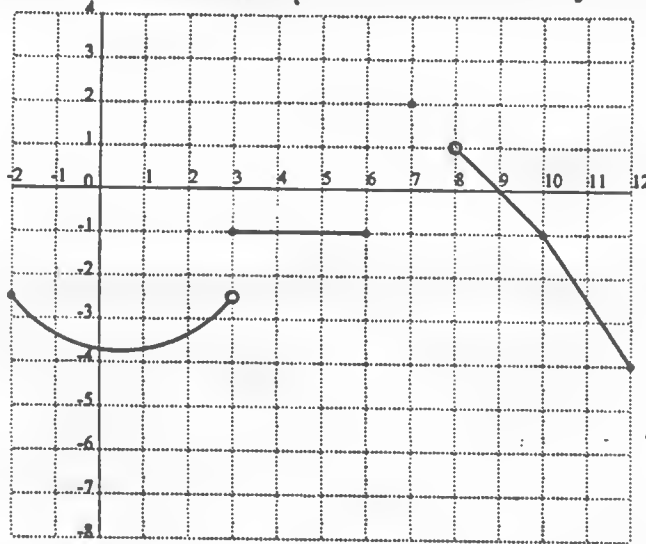
a) Assuming that the domain of f is a subset of $[-2,12]$, give the domain and range of f? $D_f = [-2,6] \cup 7 \cup (8,12]$

$R_f = [-4,1) \cup 2$

b) Give values (if possible) for the following or state that it does not exist.

$\lim\limits_{x \to -2^+} f(x) = -2.5$ $\lim\limits_{x \to 7} f(x) = DNE$

$\lim\limits_{x \to 3^-} f(x) = -2.5$ $\lim\limits_{x \to 8^+} f(x) = \underline{7}$

$\lim\limits_{x \to 4}(f(x) + 1)^2 = 0$ $\lim\limits_{x \to 6^-} f(x) = -\underline{7}$

$\lim\limits_{x \to 10} f(x) = -\underline{7}$ $f(8) = DNE$

$f(7) = 2$ $f(10) = -\underline{7}$

c) Determine all points a in the domain of f where f is discontinuous and justify your answer by stating which continuity condition fails. State the interval(s) in the domain of f such that f is continuous on each interval and such that each interval is as large as possible.

-2

pts $3, 6, 7, \textcircled{8}$ ① all are in domain   6,7,8

Because $f(x)$

[2]   2. Consider

$$F(x) = \sqrt{\frac{1}{e^x + 1}}$$

a) Find functions f, g, and h such that $F(x) = f(g(h(x)))$

$\boxed{-2}$

b) What is the domain of $F(x)$?

[6]   3. Find the domain of the following functions (show all your work and justify your answer):

a) $f(x) = \sqrt{\dfrac{x^2 - 4}{x - 2}} = \sqrt{\dfrac{(x-2)(x+2)}{x-2}} = \sqrt{(x+2)}$ $\left(\lim_{x \to 4} \sqrt{x+2} = \infty\right)$ ✗

- $X \pm 2$ because this gives us a number over 0 which is undifined. ✓
- Because the $\sqrt{}$ is allready in the question we only tche the positues.

$\boxed{\overset{0}{c}\,_c\, D_f = (2, \infty)}$

$\boxed{-1.5}$

b) $f(x) = \dfrac{1}{\sqrt{6 - x - x^2}}$ $\overset{0}{c}_c\, D_f = (-\infty, 2)$

$\lim_{x \to a} f(x)$ $\lim_{x \to 2^{\pm}} \dfrac{1}{\sqrt{6-x-x^2}} =$ $\lim_{x \to a} \dfrac{1}{\sqrt{6 \cdot x - x^2}} = -\infty$

$\boxed{-3}$

x	y
2.1	3.29
1.9	4.29

1	9
1	1.9
$\frac{1}{-1}$	$\frac{17.1}{19.6}$
0	
$\frac{10}{-1.41}$	$\frac{7.61}{}$

2.31 $6 - 2.31 = 3.69$

1.71 $(-1.71 = 4.29$

$x = 2$

[2]   4. Find all vertical asymptotes of $f(x) = \dfrac{x^2 + x - 6}{x^2 - 9}$ and justify your answer by finding all relevant limits. $\dfrac{(x+3)(x-2)}{(x+3)(x-3)}$ only posibilities at 3, ✓

$\lim_{x \to 1} f(x)$ $\lim_{x \to 1} f(x)$

[10]   5. Evaluate the following limits.

a) $\lim\limits_{x\to -3+} \dfrac{x^2+2x-4}{(x+2)^3}$ $\dfrac{-1,1}{-1.1}$ $= \boxed{1}$ ✓

X	y
-2.9	+1.1
-2.99	+1.01
-2.999	+1.001
-2.9999	+1.0001
-3	1

b) $\lim\limits_{x\to -2-} \dfrac{x^2+2x-4}{(x+2)^3}$ $= \dfrac{-4}{-.001}$ $= 4000$ $= \boxed{\infty}$ ✓

X	y
-2.1	4000
-2.01	000
-2.001	
-2.0	DNE asymptote

c) $\lim\limits_{x\to -1} \dfrac{2x+2}{x^2-x-2}$ asymptote at $x=-1$ ✗ $\boxed{DNE}$

$(x+1)(x-2)$

X	y
-1	DNE
-1.1	7
-.9	$\frac{-2}{.29}$

$1.1^2 + 1.1 - 2$ $2(-9)+2$ $=-1.8+2$ $= .2$ $\dfrac{-.2}{.29}$

$1.1 + 1.1 - 2$ $.81+.9-2 = -2+1.71 = -.29$ $= \dfrac{.2}{.29}$

$.2$ $= 7$

approching diff #   No Limit   or $\boxed{DNE}$

d) $\lim\limits_{t\to 3} \dfrac{\sqrt{t+6}-3}{3t^2-27}$ $= \dfrac{\sqrt{9}-3}{3(3)^2-27}$ $= \boxed{0}$

$9?$

$81-27$

[5]   5. Consider the following piecewise defined function:

$$g(x) = \begin{cases} x^2 - 2 & x \le 3 \\ x & x > 3 \end{cases}$$

a) Prove that g is discontinuous at $x = 3$.

① $\lim_{x \to a^+} g(x) = g(a)$

$\lim_{x \to 3^-} x^2 - 2 = 7$

② $g(a) = g(3) = 7$

$\lim_{x \to 3^+} x = 3$

① 3 is in domain
$x \le 3$

③ $g(a) = 7 \ne 3$ So it is not Continuous

b) Sketch a graph of g.
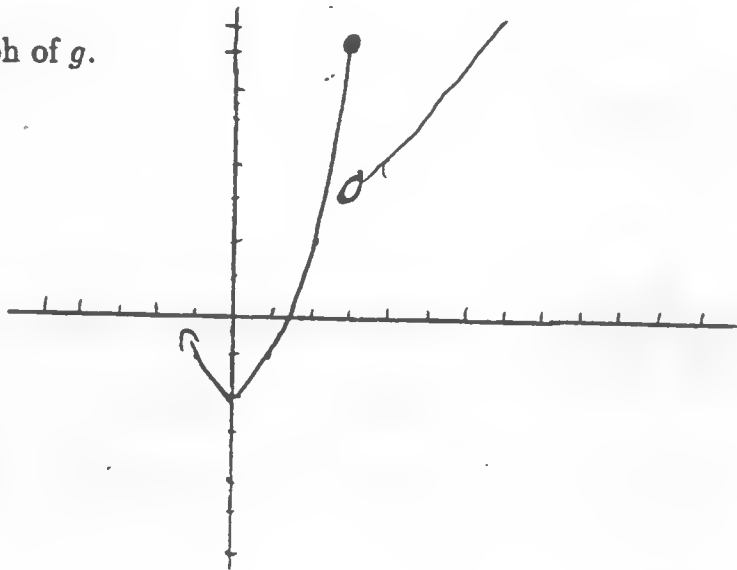
c) Consider the following piecewise defined function:

$$h(x) = \begin{cases} x^2 - 2 & x \le 3 \\ x - k & x > 3 \end{cases}$$

For what value(s) of k is $h(x)$ continuous at $x = 3$? Justify your answer.

$h(x) = x - H$ $h(x) = 7 \longleftarrow 3^2 - 2 = 7$
                                    $x = 3$
$7 = 3 - H$